Problem 11.3

Given: $\vec{A} = \hat{i} + 2\hat{j}$ and $\vec{B} = -2\hat{i} + 3\hat{j}$

a.) Determine: $\vec{A} x \vec{B}$

Minor note: The *cross product* between two vectors produces a third vector whose direction is perpendicular to the plane defined by the original vectors. In this case, both of the original vectors are in the *x-y plane*, which means we should expect a *cross product* in the *z-direction*. Executing the operation:

$$\vec{A} x \vec{B} = \left| \begin{array}{ccc|cc} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 1 & 2 & 0 & 1 & 2 \\ -2 & 3 & 0 & -2 & 3 \end{array} \right.$$

$$= (\hat{i})[(2)(0) - (0)(3)] + (\hat{j})[(0)(-2) - (1)(0)] + (\hat{k})[(1)(3) - (2)(-2)]$$

$$= 7\hat{k}$$

$$\Rightarrow \quad \left| \vec{A} x \vec{B} \right| = 7$$

b.) Determine the angle between the two vectors.

We know that the cross product is:

$$\left| \vec{A} x \vec{B} \right| = 7$$

But approaching this from a Polar Notation perspective, we can also write:

$$\left| \vec{A} x \vec{B} \right| = \left| \vec{A} \right| \left| \vec{B} \right| \sin\theta$$

The magnitude of each vector is:

$$\left| \vec{A} \right| = \left((1)^2 + (2)^2 \right)^{1/2} = (5)^{1/2}$$

and

$$\left| \vec{B} \right| = \left((-2)^2 + (3)^2 \right)^{1/2} = (13)^{1/2}$$

so:

$$\left| \vec{A} x \vec{B} \right| = \left| \vec{A} \right| \left| \vec{B} \right| \sin\theta = 7$$

$$\Rightarrow \quad 7 = \left((5)^{1/2} \right) \left((13)^{1/2} \right) \sin\theta$$

$$\Rightarrow \quad \theta = 60.3^{\circ}$$